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14. ABSTRACT
This report contains information on the FY15 final year personnel, activities, accomplishments, and publications, of this project, which was part of the ONR Fish Acoustics Basic Research Challenge.

15. SUBJECT TERMS
Swim Bladder Fish Schools Stochastic Motion Scattering Variations.

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Mathematical modeling of space-time variations in acoustic transmission and scattering from schools of swim bladder fish (FY15 Final Report)

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LONG-TERM GOALS

The goal of this project has been the development of a time-domain acoustical method for investigating the spatial and temporal stochastic variations in fish density within fish schools, which would thereby enable the study of statistical fluctuations in the scattering of sound from these objects.

OBJECTIVES

The objective of this research has been to develop a time-domain theory of acoustic scattering from, and propagation through, schools of swim bladder fish at and near the swim bladder resonance frequency, including multiple scattering and coherent interaction effects between the fish. The aim has been to develop a prescriptive capability for modeling the evolution of sound pulses as they are scattered from, and pass through, fish schools, and to develop an enhanced understanding of signal scattering, and extinction, by the school, and the fluctuations in these properties.

APPROACH

The personnel participating in this work during FY15 were: Principal Investigator: Christopher Feuillade - Ph. D. (Physics), Manchester, UK, 1977. (Visiting Professor, Pontificia Universidad Católica de Chile); Assistant: Maria Paz Raveau - Civil Engineer in Sound and Acoustics - INACAP, Chile, 2009. (Doctoral student, Pontificia Universidad Católica de Chile), Simon Alfaro Jimenez (Civil Engineer in Sound and Acoustics - INACAP, Chile, 2015), Jorge Antonio Cellio (Civil Engineer in Sound and Acoustics - INACAP, Chile, 2015).

(1) This work was developed from a scattering solution for a fish school described in 1996 (Ref. 1), based upon the harmonic solution of sets of coupled differential equations, each describing scattering from one fish. The Love swim bladder model is used as the scattering kernel (Ref. 2).

Solutions are obtained by solving a matrix equation $\mathbf{Mv} = \mathbf{p}$, where $\mathbf{v} = \{\overline{v}_1, \ldots, \overline{v}_n, \ldots, \overline{v}_N\}$ and $\mathbf{p} = \{-P_1 e^{i\phi_1}, \ldots, -P_n e^{i\phi_n}, \ldots, -P_N e^{i\phi_N}\}$ are column vectors containing the steady-state volume

oscillation amplitudes for the individual bladders, and the external fields applied to them (where P_n and ϕ_n are the amplitude and phase of the external field incident on the n-th fish swim bladder), respectively. If there are N fish in the school, $\mathbf{M}$ is an $N \times N$ matrix with elements:

$$M_{nn} = \kappa_n - \omega^2 m_n + i\omega b_n \quad ; \quad M_{nj} = \frac{-\omega^2 \rho e^{-iks_{jn}}}{4\pi s_{jn}} \quad (n \neq j)\,. \tag{1}$$

Each diagonal term [i.e., M_{nn}] describes the resonance behavior of an individual swim bladder. The quantities m_n , b_n , κ_n , etc., are varied to allow for varying values for the individual swim bladder radii, damping, etc.. Every off-diagonal element [i.e., M_{nj}] describes the radiative coupling between two of the bladders, where s_{jn} denotes the separation between the j-th and n-th bladders, etc.

The solution $\mathbf{v} = \mathbf{M}^{-1}\mathbf{p}$ describes steady-state scattering from the whole ensemble as a function of the external field amplitude and frequency. Once the solutions $\overline{v}_n$ are found, the total scattered pressure field for the whole school, for any azimuthal angle, is given by coherent summation, i.e.,

$$p_s = -\frac{\rho\omega^2}{4\pi}\sum_{n=1}^{N}\frac{\overline{v}_n e^{-ikr_n}}{r_n} \approx \frac{P_0}{r}\,\mathbf{f}_s \implies \mathbf{f}_s(k,\theta,\phi) = \frac{-\rho\omega^2[\,\sum_{n=1}^{N}\overline{v}_n e^{-ikr_n}\,]}{4\pi P_0}\,, \tag{2}$$

where r_n is the distance between the n-th swim bladder and a point receiver in the far-field. For a school containing multiple fish, the ensemble scattered field will be affected by coherent interactions between the scattered fields from the individual fish. For this reason, the phase factor e^{-ikr_n} for each swim bladder is included inside the summation of Eq. (2). The right hand equation of Eq. (2) defines a scattering amplitude $\mathbf{f}_s$ for the whole school. The scattered pressure field can be obtained for a receiver placed at any arbitrary orientation with respect to the fish school, and for any bistatic angle with respect to the acoustic source.

The steady-state volume oscillation amplitudes $\overline{v}_n$ are initially defined, in the original coupled equations, to include all radiative interaction (i.e., multiple scattering) processes between the swim bladders. The use of the $\overline{v}_n$ (via $\mathbf{v} = \mathbf{M}^{-1}\mathbf{p}$) to calculate the total scattered pressure of the school, by Eq. (2), thereby inherently incorporates modifications of the scattered pressure due to multiple scattering.

Back scattering from a school is typically represented by the target strength, which varies with frequency ω, and is related to the scattering amplitude by:

$$\mathrm{TS}(\omega) = 20\log_{10}|\mathbf{f}_s(k,\pi,0)|\,, \qquad [\mathrm{dB}] \tag{3}$$

where $\mathbf{f}_s(k,\pi,0)$ is the school scattering amplitude evaluated in the back scattering direction (i.e. in the direction counter to the incident field).

(2) Time-domain solution for a school: Here, the aim is to obtain analytic time-domain solutions of a more generalized form of the coupled differential equations for arbitrary time-dependent external input fields, i.e., $[P_1(t),\cdots,P_n(t),\cdots,P_N(t)]$, which are not necessarily harmonic. In order to achieve this, it is necessary first to determine the impulse response of a school of bladder fish, which requires a time domain extension of the steady-state approach used previously, i.e.,

$$m_i\ddot{v}_i + b_i\dot{v}_i + \kappa_i v_i = -\delta(t-t_i) - \sum_{i\neq j}^{N}\frac{\rho}{4\pi r_{ij}}\ddot{v}_j(t-t_{ji}), \tag{4}$$

where $\delta(t-t_i)$ represents an impulse arriving at $t=t_i$ to the i-th fish, and the coupling term $\sum_{i\neq j}^{N} \frac{\rho}{4\pi r_{ij}} \ddot{v}_j(t-t_{ji})$ is the coherent summation of the pressure fields radiated by the remaining N-1 fish within the school. Note that the coupling term $\ddot{v}_j(t-t_{ji})$ includes the time delay t_{ji} between each pair of fish. The coupled system described in Eq. (4) can be written in state space. If the interaction term is left out of the system, the remaining equation takes the form of a linear system which can be solved by conventional methods. If the interaction term is then considered as an external perturbation, solutions to Eq. (4) can be determined computationally using perturbation theory (Refs. 3, 4).

(3) Dynamic school modeling: In order to test and validate the steady-state and time-domain formalisms just described, a dynamic model of fish schooling behavior has been implemented, based upon biological principles. In order to incorporate accurately the acoustic interactions between fish, the relative locations of the individual fish within the school are required as an input. To provide a realistic description of time-fluctuating levels of scattering from schools, a self-organizing model of group formation in three-dimensional space has been developed, based on biological principles of collective animal behavior (Couzin et al., Ref. 5). In this model, organization within the school is a function of alignment, and repulsive and attractive tendencies based upon the position and orientation of the individual fish. The results of using this model to simulate the fish behavior demonstrate the spatial and temporal dynamics of the fish school, and indicate how these influence the statistical variability of the acoustic scattering response as a function of frequency.

WORK COMPLETED

(1) During FY15 we have completed and implemented a novel and powerful time-domain modeling technique for the solution of the coupled equations described by Eq. (4), for a cloud of air bubbles in water (which is directly applicable to a school of bladder fish) based on perturbation theory. The differential volume $v(t)$ for each bubble was calculated as the solution of the coupled system Eq. (4), using a perturbation series, i.e. $v(t)=\phi(t)=\phi_0(t)+\phi_1(t)+\phi_2(t)+\ldots$, where the functions ϕ_n were calculated via the iterative solution of a matrix equation derived from Eq. (4). The theory and computational technique have been successfully applied to the analysis of data obtained from a field experiment.

(2) During FY15 we have completed and implemented a dynamic model for predicting time-evolving fish schooling behavior, based on up-to-date biological modeling of collective fish movements, following the animal ensemble dynamics work of Couzin et al. (Ref. 5). This has already been coupled to our current steady-state school scattering model and, in ongoing work, will be coupled to the new time-domain scattering model [Item (1) above]. The two main rules for describing the collective behavior of fish schools have been accurately implemented: (a) the fish always try to keep a minimum distance between themselves and their nearest neighbors (to avoid bumping into each other); (b) when the fish are not performing an evasive maneuver, they try to maintain alignment with their neighbors, to prevent the ensemble from dividing and separating. The work completed demonstrates computational predictions for the movement of each fish in the school as a function of time, and that the evolution of the entire ensemble configuration is determined by the behavioral constraints.

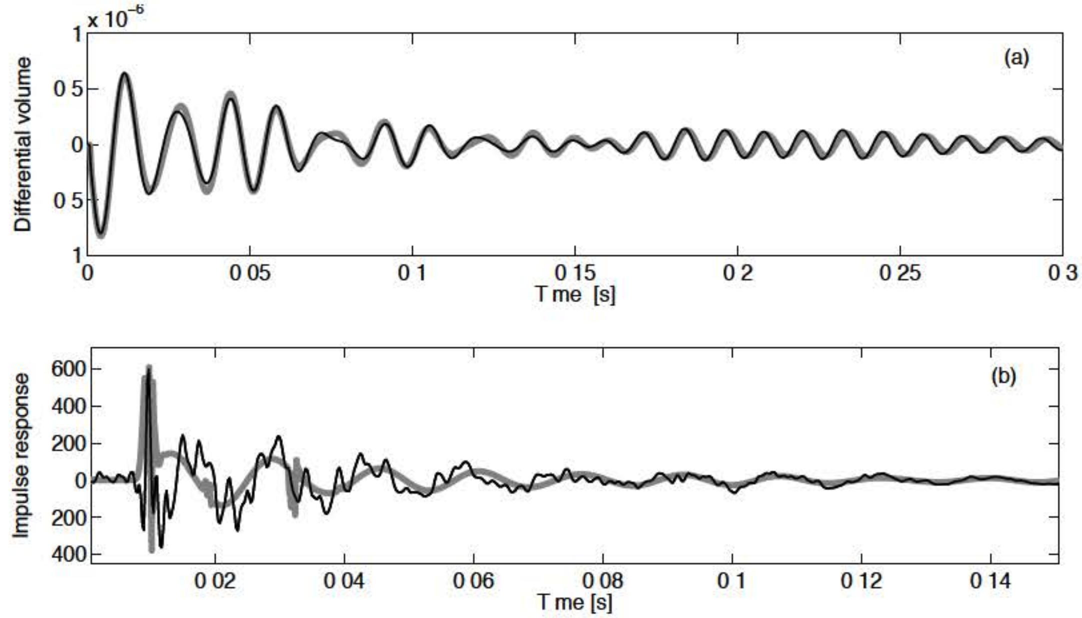

Figure 1: (a) Theoretical comparison between numerical benchmark (black line) and the perturbation-based solution (gray line). (b) Pressure impulse response due to the bubbles, for the receivers located at 6 m depth. Black line: IFFT of transfer function data. Gray line: perturbation-based solution.

RESULTS

(a) Time-domain solution of coupled equations and data analysis

The iterative technique for solving the coupled equations in the time domain, outlined above, was implemented and compared with a numerical benchmark, which includes all the multiple interactions and time delays. The numerical benchmark was implemented to solve directly Eqs. (4), using a fourth order Runge-Kutta algorithm. Figure 1(a) shows the impulse response for the differential volume of one bubble. A reasonable agreement between the numerical benchmark and the perturbation-based solution is observed from this result. The model was also tested against the transfer functions measured in Lake Travis for a group of fixed balloons. The experimental impulse response was obtained by performing an inverse Fourier transform on the measured transfer function. Figure 1(b) shows the pressure impulse response of the bubble system, for a receiver located at 6 m depth. The measured data shows a fundamental frequency that is very similar to the modeled fundamental frequency, as well as the general amplitude and other transient features that match between measurements and model. In addition, the measured data shows some other higher frequency components, superimposed with the fundamental frequency, that are not present in the model and may be caused by boundary reflections.

(b) Dynamic school modeling

The dynamic fish school behavioral model developed in this work considers three behavioral zones: the repulsion zone (*zor*), the orientation zone (*zoo*), and the attraction zone (*zoa*). The three zones are represented as spheres centered at the origin. Also, fish have a blind angle, which is represented by the conical volume shown behind it. If there are neighbors within this volume, they are not incorporated in

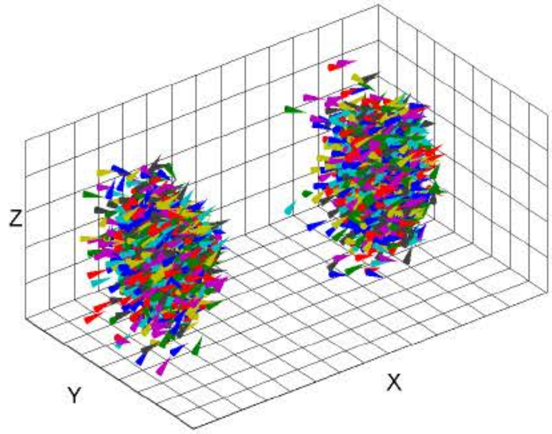

Figure 2: Dynamically Parallel school of 1000 fish. Two instants of the total simulated time are represented here, to show the general school behavior.

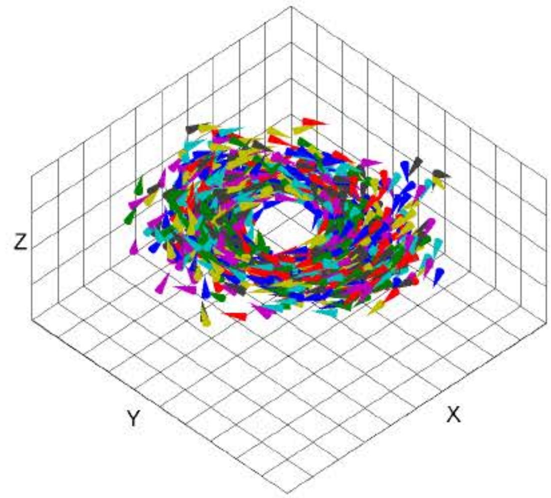

Figure 3: Torus of 1000 fish, rotating on a random direction around an empty core.

the direction calculation. The zones are represented by their radii: Δr_r, Δr_o and Δr_a, respectively, and by changing the magnitudes of these zones it is possible to model different behaviors.

Four distinct types of behaviors were found using the model described above. Using the terminology of Couzin *et al* [refCouzin], these are: (1)*Swarm*: this is a cohesive group, with low parallel orientation among its members. This occurs when individuals perform attraction and repulsion behaviors, with little or no parallel orientation; (2) *Torus*: when Δr_o is relatively small, and Δr_a is relatively large, the fish form a Torus (Fig. 3). In this case, the parallel orientation is low and the fish rotate around an empty core, where the axis of rotation is random; (3) *Dynamically Parallel group*: this configuration is much more mobile than both the swarm and the torus, and occurs at intermediate values of Δr_o, with intermediate or high values of Δr_a (Fig. 2). Here the fish school perform a rectilinear movement, but the fish still interweave with each other; (4) *Highly Parallel group*: as Δr_o increases, the group self-organizes into a highly aligned arrangement with a overall rectilinear movement. It is important to emphasize that in every case investigated, the initial school shape was spherical, and then evolved into these behaviors depending on the radii of the repulsion, orientation and attraction zones.

A simple simulated experiment was developed to model the backscattering from 3 different school types: the Torus, the Highly Parallel group, and the Dynamically Parallel group. Each school consists of 1000 fish with an average nearest-neighbor spacing $s = L = 40$ cm, where L is the length of the fish. Since it is not possible to find a stable nearest-neighbor separation for the Swarm configuration, this behavior was not included in the experiment. The sound source was placed on a ship, and the fish school was initially placed 50 m below the ship. The static school (the Torus) remains in this position for the entire simulation. However, in the cases where the schools have a displacement in space, these are considered to move rectilinearly away from this location as a function of time, keeping the same depth. The target strength was calculated for 500 time steps for each behavior type, equivalent to 50 seconds of real time, with a time step of 0.1 s, which corresponds to the response latency of a fish.

Two spectra are shown in Figure 4, which show the average target strength for 500 time steps for two behavior types, in every case for 250 fish. The target strength forms a flat plateau, probably due to the

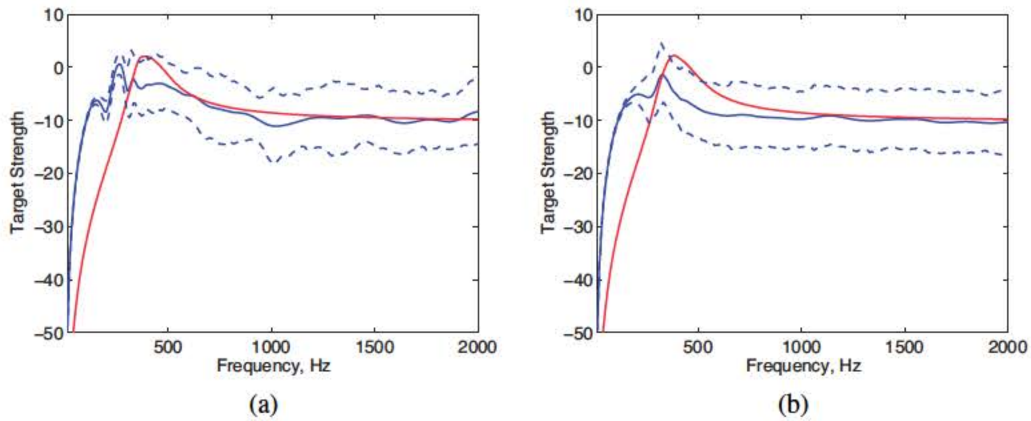

Figure 4: Target Strength for a school of 250 fish. (a) Dynamically Parallel group, (b) Torus. In each case, the blue curve shows the mean target strength for 500 time steps, when interactions between the fish are included via the fish scattering model. Confidence intervals are also shown (dashed blue line). The red curves represent the target strength obtained by incoherently summing the scattering cross sections for the individual fish.

greater time variability of the TS in the Dynamically Parallel and Torus cases, which average out to give a flatter mean value. In the Torus case, for the same number of fish, we see a higher school resonance frequency than for the parallel-type behaviors. This is due again to the fact that, in the Torus, the fish are more highly separated, with many swimming on the other side of the empty core, thus reducing the acoustic interactions between them.

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(4) M. P. Raveau and C. Feuillade, "Time domain investigations of acoustical scattering from schools of swim bladder fish," J. Acoust. Soc. Am. **135**, 2177 (2014) [Abstract]
(5) I. D. Couzin, J. Krause, R. James, G. D. Ruxton, R. N. Franks, "Collective memory and spatial sorting in animal groups," J. Theor. Biol., **218**, 1-11 (2002).

IMPACT/APPLICATIONS

The primary objective of this project, which is the development of a time-domain method for modeling, understanding, and analyzing stochastically fluctuating levels of acoustical scattering from fish schools, has been substantially advanced during the course of this fiscal year. In addition, the achievement of a dynamic model of fish school behavior, based upon observed biological characteristics, has enabled a

method for predicting the structure and motion of these objects, which are the primary causes of the observed variations in scattering. Taken together, these two aspects provide a completely new capability for predicting the statistical variations of scattering from different configurations of fish ensembles, which is of immediate importance in SONAR applications, both for detection and classification purposes.

RELATED PROJECTS

Experimental and other work by other participants in the ONR BRC, Fish Acoustics program.

PUBLICATIONS

(1) M. P. Raveau and C. Feuillade, "Sound extinction by fish schools: Forward scattering theory and data analysis," J. Acoust. Soc. Am. **137**, 539–555 (2015).

(2) M. P. Raveau and C. Feuillade, "Resonance scattering by fish schools: Evaluation of the effective medium method," J. Acoust. Soc. Am. (In review).

(3) M. P. Raveau, C. Escauriaza, C. Dolder, P. S. Wilson and C. Feuillade, "Impulse scattering from clouds of acoustically coupled gas bubbles in fluids," J. Acoust. Soc. Am. (In preparation)

(4) M. P. Raveau, C. Feuillade, G. Venegas and P. S. Wilson, "Measuring the acoustic scattering response of small groups of live fish in a laboratory tank," Proceedings of Meetings on Acoustics Vol. 23, (2015), DOI: http://dx.doi.org/10.1121/2.0000075

(5) S. E. Alfaro, J. A. Cellio, M. P. Raveau and C. Feuillade, "Low frequency scattering from dynamic fish schools based on collective animal behavior modeling," Proceedings of Meetings on Acoustics Vol. 23, (2015), DOI: http://dx.doi.org/10.1121/2.0000090

(6) M. P. Raveau, "Mathematical modeling of space-time variation in acoustic transmission and scattering from schools of swim bladder fish," Doctoral dissertation, Pontificia Universidad Católica de Chile, September 2015.